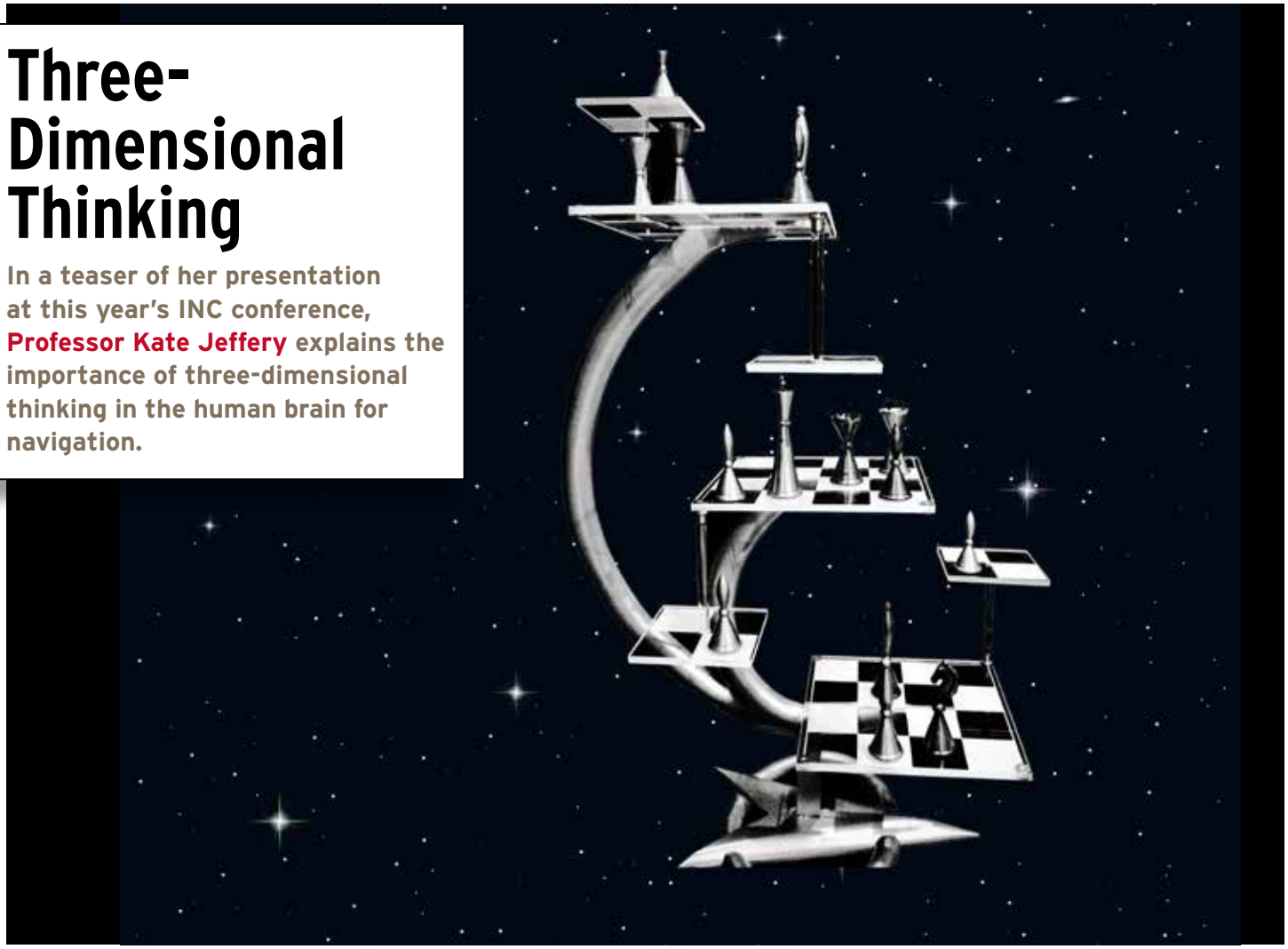
Three-Dimensional Thinking

In a teaser of her presentation at this year's INC conference, Professor Kate Jeffery explains the importance of three-dimensional thinking in the human brain for navigation.



This set and variants available to purchase. Credit: 3dchess.com.

It is a tense moment in *Star Trek II: the Wrath of Khan* as Kirk's ship the *Enterprise* and Khan's ship the *Reliant* battle it out in deep space. Spock notes that Khan's tactics suggest 'two-dimensional thinking,' since all his manoeuvres are on a plane. Kirk, his spatial reasoning honed from years of playing three-dimensional chess against Spock, exploits these constraints on Khan's thinking by surprising him with an unexpected approach from below, and wins the battle.



The power of thinking about the world in three dimensions is that it gives a more realistic interpretation of the universe. Credit: iStockphoto.com/agsandrew.

The issue of dimensions in navigation is an interesting and little-considered one. We tend in our own lives towards 'two dimensional thinking,' with our flat maps, and our tendency to airily disregard hills when planning routes using phone apps. However, the world is not two-dimensional, and taking the third, vertical dimension into account is vital when trying to navigate effectively. There is little point for example in planning a straight route from Bangor to Barmouth but to know this, one needs to know something about the topography of the land in between (and that it includes Mount Snowdon).

As a neuroscientist researching how the brain represents space, I am especially interested in the question of how it represents 3D space. This problem to be solved is different for different species – rats live in complex branching burrow systems underground, polar bears live on flat Arctic tundra, sheep live on undulating hills, monkeys live in the canopies of rainforest, dolphins swim through the sea and bats and birds fly through the air. As humans, we do all of these things, and more – recently we have begun to float through space *(Figure 1A)*, and to explore virtual reality worlds that offer even more interesting possibilities – four dimensional spaces like the virtual reality maze in Figure 1B, or impossible spaces like Escher's crazy staircase worlds (Figure 1C). How does the brain make sense of all this complexity?

We know from several decades of brain research that the brain is able to form a flat map using simple and intuitive building blocks [1][2]. Place cells, recorded from the brains of exploring rats *(Figure 2A)*, become active when the animal goes into particular regions of space, and thus seem to form a pixel-map of space. We know from virtual reality studies of human epilepsy patients, who have received implanted electrodes for diagnostic purposes, that this is true for humans too[3]. Place cells thus form a 'you are here' signal. Head direction cells *(Figure 2B)*, from nearby regions in the brain, become active when the animal's head faces in a particular direction, and seem to be a 'you are facing this way' signal. And the more recently discovered grid cells *(Figure 2C)*, also nearby to both place and head direction cells, become active in regularly repeating, evenly spaced regions of the environment and seem to be a 'you have

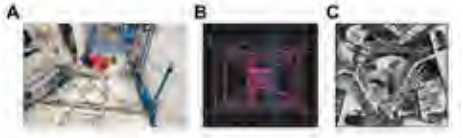


Figure 1 Crazy spaces for humans. (A) In microgravity, astronauts can float at will directly to any part of a cluttered interior space – but which way is 'up'? (B) The structure of a four-dimensional virtual-reality maze. Will we learn to comprehend four-dimensional space, if exposed to it as children? (C) Relativity by MC Escher, 1953. How does the brain make sense of spaces such as this?

walked this far' signal. All of these signals are useful tools for map use, and so it makes sense that the brain has these – the question is, do they work in vertical space too, or is their 'map' only flat? And if it is only flat, how do we know how high we are, or how to plan a shortcut across, say, a space station? *Can* we even do these things?

In order to study the brain's 'three-dimensional thinking,' we need to somehow get animals into three-dimensional space. Neuroscientists have taken two approaches to this: one is to take the animal we know best, the heroic lab rat, and to persuade it to venture off the ground and into the air. Since rats don't swim, fly or float, this is something of a challenge, but it is one that my lab has embraced in the past few years. The other alternative is to take a related animal that does fly through space, the bat, and see what its place, head direction and grid cells do.

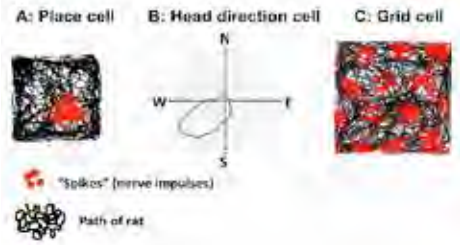


Figure 2 Spatial cells forming a flat map. (A) the activity of a place cell, recorded from the brain of a rat exploring a square box for 10 minutes. The path of the rat is shown by the black line, and every time the cell fired a nerve impulse (or 'spike'), the location of the rat at that moment is marked by a red square. Note that the cell produces all its spikes when the rat is near one corner of the box. (B) A head direction cell, which would fire everywhere in the box (not shown) but only when the rat faces in a particular direction (in this case, South-west, as shown by the polar plot). (C) A grid cell. Like a place cell, this cell fired only in restricted places – however, unlike a place cell, there are many of these places. Most remarkably of all, these places are evenly spaced, meaning the cell seems to be interested in how far the rat has walked, and in what direction.

What might these cells do? At first glance, it seems like making a map of three-dimensional space should be much like making a two-dimensional one, only a bit bigger (quite a lot bigger actually, since the size of a flat map is the square of its linear extent while the size of a volumetric map is its cube). However there are added complexities of three dimensions, over and above two. One is that rotations in three dimensions are order-dependent, which is a property that mathematicians call non-commutativity. Put simply, on a flat surface it doesn't matter if you rotate first left and then right, or first right and then left, you still end up facing in the same direction. However, in three dimensions the order of the rotations is critical, and mixing them up gives a quite different end result *(Figure 3)*. For a navigation system trying to track rotations in three dimensions, it is vital to get this right and to make sure that the detectors for each rotation plane convey their signals to the 3D compass in the correct order. In vertebrate animals (which includes humans), these detectors are fixed relative to each other, since they are embedded in the inner ear, or labyrinths, in the form of the semi-circular canals (if you have ever suffered the misery of labyrinthitis you will appreciate how important this function is). Some human-made navigation systems have attempted a different way of tracking rotations, in which the detectors are gimballed gyroscopes and can rotate independently of each other. This arrangement capitalises on the way gyroscopes stay locked to the universe's reference frame rather than rotating with the substrate they are on, but has a fatal flaw which is that sometimes two of the detectors will accidentally end up in the same plane, a quandary known as gimbal lock, which reduces the detectable planes of rotation from three to two. A famous example of gimbal lock occurred on the Apollo 11 moon mission during manoeuvres in space, with fortunately (in this case) benign consequences[4]. The bottom line is, whichever rotation detection system you use, tracking 3D rotations is hard, and computationally demanding.

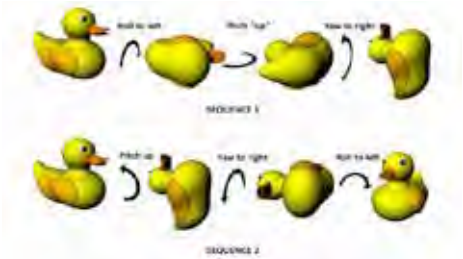


Figure 3 In three dimensions, rotations are non-commutative (order-dependent).

If navigation occurs over a surface, instead of through a volume, there are two more intriguing challenges for a 3D compass. The first is the unfortunately named hairy-ball problem, a well-known problem in mathematics which is how to comb a hairy ball completely flat without leaving crowns, or 'singularities' *(Figure 4A)*. This turns out not to be possible. If the hairs represent compass headings, as might be produced by head direction cells for example, one can see that there will always be places where the vectors abruptly reverse direction. An animal that was facing North on the North face of the sphere will suddenly, as its nose tips past vertical, finds itself facing South, and the head direction cells have to abruptly change their state. How does a brain compass deal with such sudden, disorienting transitions?

There is one more problem faced by a 3D compass and this is one discovered by, oddly enough, quantum physicists. Many subatomic particles have a spin, which means that one end differs from another, since the rotations go clockwise around one pole and counter-clockwise around the other. This polarisation means that each spinning particle is like a tiny compass. Berry and Hannay independently realised that if a spinning particle was translated on a curved path, such as over the surface of a sphere, then the direction of its spin – its compass direction – would be altered by the time it got back to its starting position, even though the particle was never explicitly rotated (*Figure 4;* see[5] for review). This is because it was *im*plicitly rotated by virtue of moving through three dimensions instead of only two. This raises an interesting question – if a rat walks over the surface of a sphere, and never actually rotates in the plane of its head/body, which is the normal condition under which head direction cells rotate their signal, then will the rat's internal compass be pointing the wrong way when it gets back to its starting point *(Figure 4B)*? We think the brain is clever enough to detect and compensate for this occult rotation, since bats appear to have had directions cells that point in any direction is 3D space[6], and rats appear to generate head direction activity which, while appearing only two-dimensional, compensates for 3D rotations[7]. Are humans more like rats, or more like bats? We don't know yet, but it will be interesting to find out.

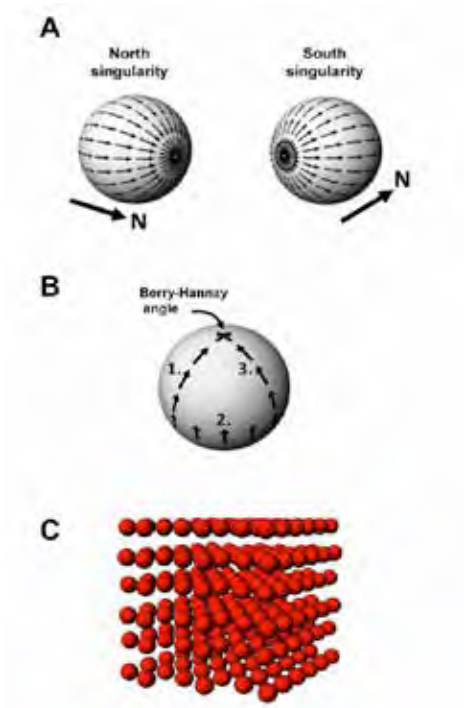


Figure 4 Two problems faced by a 3D compass on a curved surface. (A) The hairy ball problem: it is not possible to smoothly carpet a sphere (or any closed, curved surface) with hairs, or – in this case – vectors – there will always be two places where they meet ('crowns' or 'singularities'). (B) Berry-Hannay errors occur when a vector is transported over a curved surface – in this example the arrow was transported on a three-legged journey – although it was never explicitly rotated, it acquired a rotation due its curved path through space. How does a 3D compass detect and correct for these errors? (C) Would a grid cell do this – fire in a three-dimensional lattice pattern – if a rat could move freely through space?

Navigation requires a map and a compass – the compass faces the challenges we discussed above, but a 3D map faces a challenge, too.

As noted earlier, a 3D map is much bigger than its 2D counterpart, and brain tissue is enormously costly for an animal to build and maintain, so it would be efficient to use only a 2D map if that would suffice for most things. There is also the added complexity of movement tracking. To plot movement through space, using a map, one needs to measure the distance walked in a given direction – so can the brain track distance moved in vertical space as well as horizontal? Put another way, do grid cells make their grids vertically as well as horizontally *(Figure 4C)*? In my lab we are currently trying to find out, using climbing rats – preliminary indications are that if it does so, it does it with less precision, but the acid test will be when we can persuade the animals to move through a volumetric space while we record their grid cells. The bat and rat neuroscientists are in a race to answer this question. Again, it may turn out that bats and rats are simply different.



Rats! Is there a reverse gear on this body? Credit: iStockphoto.com/Iculig.

Why do we care about three dimensional thinking? The main reason – apart from simple curiosity – is that as a species, we are venturing into new spaces offering new dimensional opportunities.

Space flight is one example – astronauts find themselves moving through space in ways that were never intended by nature, with often interesting effects. For example, they frequently report reorientation episodes, where they float away from one surface, which they had thought of as the 'floor', towards the opposing surface – the 'ceiling' – that suddenly reverses identity and becomes a new floor, creating an experience of sudden flipping upside down[8].

Designing space station interiors that allow efficient use of space while remaining 'coherent' to the users of that space will be increasingly important, particularly if space tourism ever takes hold (reorientation episodes are nauseating and upsetting – not a great selling point for a space holiday).

Even more compelling is the technology offered by virtual reality, which is becoming ever more sophisticated and can allow spaces never found in nature, such as four-dimensional spaces (see[9] for an interesting example of navigation in a 4D maze) or impossible spaces.

What will happen to children who grow up exploring these new worlds? Will they, like Captain Kirk, be able to train their spatial awareness with practice, perhaps even to beyond the limits we can currently achieve? It will be interesting to see.

References

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Get Your Kirk On And Stretch Your Mind To Three Dimensions



Three-Dimensional chess is a growing hobby for those who laugh in the face of Sudoku, and for whom the Times cryptic crossword is a mere bagatelle.

As well as offering clear advantages in terms of defeating augmented warlords in space battles (should the need arise – you never know), keeping the mind active is a well-respected way of staving off conditions like memory loss and logic degradation, so an occasional game of 3D chess could be an excellent idea for navigators keen to keep their hand in.

The first thing you'll need is a board, because, as you can see, 3D chess is quite literally a whole different board game to ordinary chess. Many companies exist that will sell you (or indeed make you, to your colour choice) a 3D chess board, including our friends at 3Dchesset.com. The main difference is that a 3D chess board is split into three different 'main boards' on three different levels, and then also includes personal 'attack boards' that can be moved around throughout the game, either above or below the main playing surfaces.

Essentially, the top level is 'Black's Board,' the bottom level is 'White's Board, and the middle level if the 'Neutral Board' – equivalent to the lines of blank space between the two opposing armies on a two-dimensional board.

For a full set of 3D chess rules, go to http://3dchessset.com/location/ - stretch your mind today, and join the 3D revolution.